Models for the evaluation of routing and machine flexibility

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Abstract

In this paper we are presenting models by which flexibility for a manufacturing system can be assessed. Two of the most fundamental types of flexibility namely routing and machine flexibility, are examined. These models enable a manager to compare different system designs. Furthermore, these measures provide a manager with a tool to evaluate the ongoing operations of a system over time and over different conditions. These models reflect the view that the flexibility of a system is a function of the technology as well as of how well the system is managed. The efficacy of these methods is demonstrated through numerical examples.

Key Words: Manufacturing System, Routing, Machine Flexibility

1. Introduction

Flexibility

Flexibility of any manufacturing system is its capability of processing a variety of different part styles simultaneously at the various work station and mix of part styles and quantities of production can be adjusted in response to changing demand pattern.

Requirements to be flexible

- Ability to identify and distinguish among the different part or product styles processed by the system.
- Quick changeover of operation instructions.
- Quick changeover of physical setup.

Need for flexibility

Now, with product life cycles becoming more compressed, firms are looking for a way of extending the design life of their plant in order to get more their capital investment.

Flexibility, along with cost, quality and service is an important aspect of manufacturing strategy. Through- out most of the industrial era a great deal of attention was focused on the cost component in production. In the 1970's and 1980's, as a result of increased Japanese competitiveness, quality was the factor, which came into the limelight. Now, with product life cycles becoming more compressed, firms are looking for a way of extending the design life of their plant in order to get more from their capital investment. As a result of this change in the market, together with the advent of new manufacturing technologies referred to as 'Flexible Manufacturing Systems', flexibility is receiving more notice. The aim of this investigation is to provide methods for the evaluation of certain key types of production flexibility. Since flexibility is a function of the machine sequence and operation, these methods will enable managers to compare different manufacturing system designs.

Browne et al.'s (1984) taxonomy breaks flexibility down into eight classes: routing, machine, process, product, volume, expansion, operation, and production flexibility. Routing flexibility gives the system the capability to continue producing a given set of parts despite machine breakdown. Machine flexibility is the ability to easily make changes to a given set of parts. Browne et al. state that process and product flexibility are dependent on machine flexibility. They also assert that volume, expansion and operation flexibility are dependent on routing flexibility (production flexibility being a function of all the other seven types). Thus, the natural starting points to develop evaluation procedures for manufacturing flexibility would be routing and machine flexibility.

Chatterjee et al. (1984) offered four different measures for routing flexibility. These measures ere: (i) the cardinality of the set of routings, (ii) the ratio of the number of module centers capable of carrying out an operation on a certain part to the total number of module centers, (iii) the number of alternative parts within a module center, and (iv) the possible trajectories through the module centers. Chung and Chen (1989) also provide a measure of routing flexibility in a similar spirit to that of Chatterjee et al. This value is in terms of the reduction in lead time and is given by the fractional decrease in the total job makespan using alternative routes. The above measures of routing flexibility increase with the number of machines capable of processing the part without considering the reliability of the machines. Thus a highly reliable two machine is considered by these measures to add as much flexibility as a less reliable machine. Yao (1985) presents a measure of routing flexibility, which is based on the concept of entropy and includes the reliability of machines. Another factor neglected in the above measures is that of machine capacities, as machines with differing capacities are weighted equally under their framework. The measure that we propose takes both of these factors into account and can thereby capture the interaction between these two factors. A number of researchers have suggested promising measures for various types of flexibility but have not provided guidance as to how one should compute these measures.

These measures of manufacturing flexibility {cardinality of the route set, rate flexibility, and entropy) are difficult for managers to interpret. Our procedure demonstrates how operations performance can translate into economic measures relevant for managers.

2. Routing flexibility

In this section method for appraising routing flexibility is proposed. Routing flexibility is exhibited when machines break down. As a result we incorporate the reliability of machines in our models. The system designed consists of machine centers and the materials handling system. The design specification includes the reliability of the different machines, their capacities for each part type, and the precedence relationships. Reliability is defined as the probability that the machine is capable of performing an operation at a given time.

Capacity is defined as the total number of units of a part type a machine can process in a given block of time. The procedure results in the computation of an expected maximal cash flow for a given production system design and product mix which is an economic measure representing routing flexibility. Consider a manufacturing system consisting of a number of machining centers and a materials transfer system. This network can be represented by a random, planar graph where the vertices (machining centers) are subject to failure. We assume that the failures are independent. The following parameters and variables used are:

- $.i$ is the index of the part type; $i = 1, \ldots, m$.
- $.k$ is the index for machine type; $k = 1, \ldots, n$.
- $.h$ is the index of an elementary path from the load to the unload station; $h = 1, \ldots, H$.
- $.t\ ik$ is the number of time units required to process one unit of part i on machine k.
- $.T\ k$ is the total number of time units available for processing at machine k.
- $.Pk$ is the probability that machine k is operating at a given point in time.
- $.bikh$ is a zero-one parameter which if equal to one indicates that product i can be produced on machine k on path h.
- $.ailk$ is the element of the arc-incidence matrix for product i indicating a connection between machine l and k. The element is one if a connection exists and zero otherwise.
- $.Cih$ is the contribution margin of part type i processed on path h, calculated as the price of part i less the cost of processing on path h and less the raw-materials cost. Note that the cost of processing depends on the reliability of the machines on the path for that part.
- $.di$ is the minimum demand that must be satisfied for each part type I
- $.Xih$ is the flow of part i on path h.

The performance measure to reflect the routing flexibility (RF) of the manufacturing system should combine both the cardinality of the route set and the reliability of the system. **The measure is the maximum expected contribution of the system**. Such a measure translates operational differences of systems into financial terms, which would be of greater use to managers evaluating various designs.

Path formulation

$$RF = \text{Max} \sum\sum c_{ih} x_{ih}$$

Subject to

$$\sum_{i} \sum_{h:bihk=1} \frac{t_{ik}}{p_k} x_{ih} \le T_k \qquad \text{for all } k, \qquad (1a)$$

$$\sum x_{ih} \ge d_i \qquad \text{for all } i, \qquad (1b)$$

$$x_{ih} \ge 0 \qquad \text{for all } i,h \qquad (1b)$$

(1a) is the capacity constraint for each machine given the reliability of that machine. *ti k/Pk* is the expected amount of time to process part i on machine k. Constraint (1b) ensures that certain minimal demand conditions are satisfied. This formulation is preferred when the network is sparse.

An alternative formulation is one which centers around the machine. In addition to the previously defined variables we specify a new variable *Yilk* to be the flow of part i from machine I to machine k (where machine 1 is the load station and machine n is the unload station). Also we define Pri to be the price of part i, and *ucilk* to be the cost of processing part i on the arc connecting machine I and machine K.

Machine formulation

$$RF = \text{Max} \sum_{I} Pr_i\, y_{inl} - \sum_{i} \sum_{l} \sum_{k} uc_{ilk}\, y_{ilk}$$

Subject to

$$\sum_{l:ailk=1} y_{inl} - \sum_{l:\ ailk=1} y_{ilk} = 0 \qquad \text{for all } k,i, \qquad (2a)$$

$$\sum_{i} \sum_{l:ailk=1} \frac{t_{ik}}{P_k} x_{ih} \le T_k \qquad \text{for all } k, \qquad (2b)$$

$$\sum y_{iln} \ge d_i \qquad \text{for all } i, \qquad (2c)$$

$$y_{ilk} \ge 0 \qquad \text{for all } i,l,k, \qquad (2d)$$

Where n is the index. for the unload station. Constraints (2b) and (2c) are equivalent to

constraints (la) and (lb), respectively. Constraint (2a) is introduced in this formulation to ensure the balance of flows, i.e. the number of units flowing into a machine is equal to the number of units flowing out from a machine. This formulation has in its worst-case *nm2* variables and *nm + n + m* constraints. Consequently this formulation is preferred when the network is dense. Both of the above formulations are LP models, which can be solved
using a standard simplex code as is done in the example in the following section.

From both of the above models it is clear that with increasing reliability *(p's)* and capacity *(T's)* of the components in the system, he expected thru put of the system is non decreasing. The model implicitly captures the phenomenon where routes with higher flow times (either due to capacity restrictions or reliability problems) will be less vorable and fewer units will be produced on this path. We now give an example to illustrate the evaluation f this flexibility measure.

An example

Let us use the procedure of the previous section to evaluate the following two manufacturing system designs. The first system has two machines and load and unload stations (Figure 1), while the second differs in that it has a third machine
Figure 1. Sample system design no.1

TABLE 1

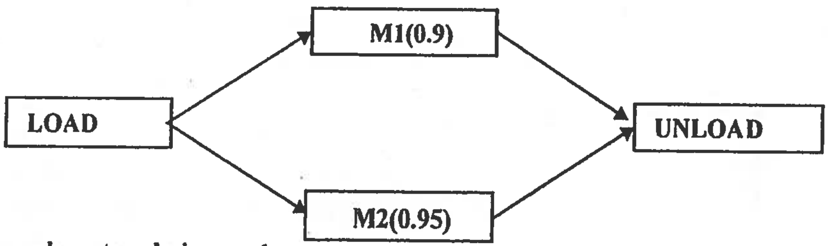

Figure 1. Sample system design no. 1

TABLE 1

PART	Processing time (t_{jk})			Contribution(Ci)
	M1	M2	M3	
1.	3.3	3.5	4.5	3.5
2.	3.5	3.4	4.9	4.0

(Figure 2) and the reliabilities of the machines are different. For ease of exposition, in the following examples a part is assumed to have the same contribution regardless of the path taken through the plant. Let *Tk=300* time units for all *k, d1 = d2* = 10 units, with the part types having the characteristics as in Table 1. The systems are illustrated in the following two figures, where the reliability of the machines *(pk for all k)* are the numbers in parentheses (i.e. the probability that machine MI is operating at a given point in time is 0.9 while for machine M2 it is 0.95).

The routes for each part type in this system are:

1. L-MI-U.
2. L-M2-U.

This system yields an optimal expected contribution of 641.2 with 10 units of part 1 processed on machine 1, 67.71 units of part type 2 being processed on machine 1 and 83.82 units processed on machine 2. The resulting volume is the capacity of the system. The design of the second system is given in figure 2.

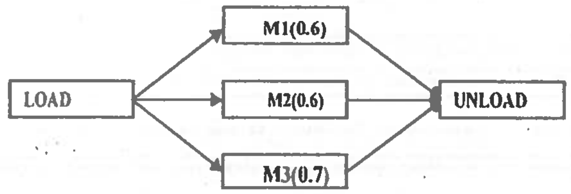

Figure 2. Sample system design no.2

The routes for each part type in this system are:
1. L-MI-U.
2. L-M2-U.
3. L-M3-U.

This system yields a lower optimal expected contribution of 551.9 with 10 units of part type 1 being processed on M3 and the processing of part type 2 being spread out on all three machines {MI producing 51.4 units, M2 producing 44.1 units, and M3 producing 33.7 units). It is clear from the above example, that the cardinality of the route set is insufficient for appraising a value of flexibility. In the example, system no.2 has a greater number of alternative routes {a higher cardinality of the route set) but is less flexible. Thus, reliability must be incorporated along with the number of routes in the evaluation of a manufacturing system. One could examine the possibility of adding more machines on to the first network by using this method.

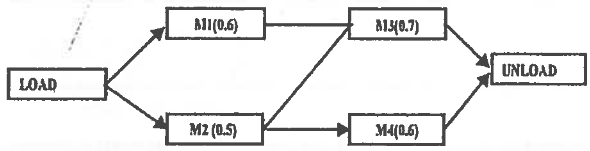

Figure 3. Sample system design no.3

PART	Processing time (t_{jk})				Contribution(Ci)
	M1	M2	M3	M4	
1.	3.3	3.5	4.5	3.8	3.5
2.	3.5	3.4	4.9	3.9	4.0

Increasing reliability, however, does not always imply an increase in contribution as the analysis of a more complex machine network illustrates. Let $T k = 300$ time units for all k, dl = $d2$ = 10 units, with the part types having the characteristics shown in Table 2.
The routes for each part type in this system (see Figure 3) are:

1. L-MI-M3-U.
2. L-M2-M4-U.
3. L-M2-M3-U.

The optimal solution in this system yields an RF of 346.16 with 10 units of part 1 produced on route 1 and 33.67 and 44.11 units of part 2 produced on routes 1 and 2, respectively. In this system M2 and M3 are bottlenecks, so that increasing the reliability of anyone of those machines would yield an increased contribution. If the reliability of M2 were increased, there would initially be an increase in RF, but beyond 0.52, M4 becomes a bottleneck and any further increase in reliability of M2 brings no added contribution.

3. Machine flexibility

Machine flexibility is dependent on the ease with which one can make changes in order to produce a given set of part types. One possible measure of this form of flexibility could be the time taken to set up the machine to perform some operation on a different part type. With a decrease in set-up time the scope of product designs produced efficiently increases, thus demonstrating how product flexibility is dependent on machine flexibility. Roller and Tombak (1990) have shown the market conditions under which this type of flexibility is desirable. In this section we propose a measure by which alternative manufacturing systems can be evaluated with respect to machine flexibility. Since machine flexibility is not only built into the design of the system but is also a function of how the system is managed, our measure can also be used for control of operations. Sethi and Sethi (1990) point to "numerical control, easily accessible programs, automatic tool changing ability, sophisticated part loading devices, size of the tool magazine, standardized tools, number of axes, etc." as sources of machine flexibility. Appropriate measure for machine flexibility is the time required to replace worn-out or broken cutting tools, change tools in a tool magazine, assemble or mount the new fixtures required, prepare cutting tools, position the part, and changeover the numerical control program. We have chosen to concentrate on the time required to change tools in a tool magazine, the time required to change the tool in the machine when the tool is in the magazine, and the time required to assemble or mount the new fixtures required (these times all include both placement and adjustment).

We do so because we believe that these factors are the most significant portion of set-up time in many cases (see Japan Management Association, 1989; Monden, 1983; Denardo and Tang, 1988).

In order to derive a model to evaluate machine flexibility let us define the following:

- .*i*, j are indices for part types; $i = 1, ..., m$; $j = 1, ..., m$.
- .*q* is the maximum number of tools that can fit in a tool magazine.
- .*Vij* is the time to position the tool in the machine from the tool magazine if the tool for part j is different from that of part *i* (this assumes that each tool can be picked from the tool magazine in the same amount of time), also *Vij* = 0 if the tool for part j is the same as that for part *i*.
- *Uij* is the time to change the fixture if the fixture for part j is different from that of part *i*.
- *s* is the time required to change a tool in the tool magazine (this is considered the same for all tools since it involves picking, placing, and returning tools to the same location).
- *bir* = 1, if part *i* requires tool *r*, 0, otherwise.
- *Zij* = 11, if part *i* precedes part j on the machine,0, otherwise.
- *Yij* = 0, if *bir* = 1 and *bjr* = 1 for any *r* € Y; 1, otherwise,

Where y is the set of tools in the tool magazine since the last tool change.

Let $m + 1$ be a dummy part to close the sequence. Also, we assume that part $m + 1$ uses a dummy tool such that *Yim+1* = Ym+li = 0 for all *i* and that *Vim+1* = *Vm+li* = Uim+1 = *Um+li* = 0.

We assume that the tool changing time is the same for all tools but the fixture changing time may be different for each part. Assuming that the machines were incapable of changing fixtures and tools simultaneously, machine flexibility (ML) can be evaluated using the following nonlinear integer programming model:

$$MF = \text{Min} \sum_i \sum_j V_{ij} Z_{ij} + \sum_i \sum_j u_{ij} z_{ij}$$

$$+ s \sum_i \sum_j y_{ij} z_{ij} \qquad (3a)$$

subject to

$$\sum_j z_{ij} = 1 \quad \forall i \qquad (3b)$$

$$\sum_j z_{ji} - 1 \quad \forall i \qquad (3c)$$

$$\sum \quad \sum z_{ji} \geq 1 \quad \forall S \subset \{1, 2, \ldots\ldots m+1\} \qquad (3d)$$

$$i \in s \quad j \in s$$

$$y_{ij} \; z_{ij} \in (0.1) \quad z_{ij} = 0 \quad \forall i, j.$$

The first term in the objective function is the tool positioning time, the second term is the fixture positioning time, and the third term is the time required for tool interchanges. For (3a), _i _j YijZij gives the number of distinct tool interchanges. The implicit assumption for this formulation of the objective function is that a certain amount of time is taken for each tool change. This is true of systems which have a large central magazine from which tools travel back and forth and it is not economical to have large local tool magazines. Constraints (3b) and (3c) force the sequence to have a unique predecessor and successor while (3d) breaks any cycle in the sequence. Note that y is dynamic and changes with each reconfiguration of the tool magazine. This reconfiguration is done by placing the next q tools demanded by the forthcoming part sequence in the tool magazine.

The evaluation of machine flexibility requires the simultaneous determination of both the part sequence and the sequence of tools. In order to focus on the set-up times associated with machine characteristics (as discussed above) and to make our analysis tractable, we assume that the sequence of parts is given. This is not unrealistic since due dates are often exogenously given to the operations manager which often restrict the manager to a certain set of sequences. Further more, a number of heuristics might be used to choose a unique sequence from this set. Simple heuristics can be used to solve the above problem. If both the time required to change tools in the magazine and the time to change the tool in the machine dominates over the time to change fixture, a reasonable heuristic would be to group the parts by tool used. If, conversely, the fixture changing time dominates, the parts could be grouped by fixture utilized.

An example

Say $Uij = 1$ for all $i != j$, and 0 otherwise, and let $s = 10$, $q = 2$, and $m = 4$. Also let $Vij = 1$ if the tool for part
i is different from part j, and 0 otherwise. The resulting problem formulation using (3a) is

$$\text{Min} \sum_{i}\sum_{j!=i} (zij + vij\, zij) + 10 \sum_{i}\sum_{j!=i} yij\, zij$$

subject to

Constraints (3b), (3c), (3d) and (3e).
Let the sequence in which the parts are to be processed be 1, 2, 3, 4, with tools required being *A*, *B*, C, and
B, respectively. Hence the *Vjj'S* are as given in the following matrix:

$$\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

where, for example, $v12 = 1$ since the tool for part 1 is *A* and the tool for part 2 is *B*, thus setting up the machine for part 2 involves a tool change. Assuming that the fixture and tool required by the first part is already in place, the above expression is then reduced to:
$\text{Min}\{6 + 10(Y12 + Y23 + Y34)\}$.
This minimization involves choosing which tools belong in the magazine at a particular state in production. We know that $b1A = b2B = b3c = b4B = 1$ with all other $bir = 0$. Let the initial configuration of the tool magazine contain tools *A* and *B*, i.e., let $y = \{A, B\}$. Then, $Y12 = 0$ since $b1A = b2B = 1$. In order to process part 3 we need to reconfigure the tool magazine, since it does not contain tool C. y is reset to $y = \{B, C\}$, since tools *B* and C are the next two tools required. Then $Y23 = 1$, since $b2B = 1$ but $b3A = b3B = 0$, and $Y34 = 0$, since $b3c = b4B = 1$. The result is one tool change removing tool *A* and replacing it with tool C at a cost of s. The objective function value, MF, would then be 16. A better solution would be the sequence 1, 2, 4, 3, which has the same number of tool changes to the tool magazine (one), yet fewer tool changes to the machine (two instead of three), yielding an MF of 15.
In the above models we have used set-up times as a surrogate measure for evaluating the effort required to make the necessary changes to produce a given set of parts. This allows for the comparison of various manufacturing system designs which are capable of producing the same part mix. The models also

give the capability of measuring the effort involved in producing various sets of parts with the same production system. On a daily basis, the MF measure helps the manager evaluate the status of the shop in terms of which machines to use so that the flexibility of the system is enhanced. If it is possible to elicit probabilities *(Pj)* of producing a part set *(i)* then one could use a model which minimized the expected MF = *jMFjpj*. These simulations will assist the manager in studying various designs for several possible sets of parts that may be produced. Based on the results, the most flexible design can be selected.

Conclusion

Several models for the evaluation of alternative manufacturing system designs with respect to routing and machine flexibility has been provided. These models facilitate the design/technology choice process by providing a link between operational performance and economic implications. Furthermore, these measures provide a manager with a tool to continuously evaluate the system based on the criteria of routing and machine flexibility. It is said that flexibility must not only be designed in, but also managed. Thus these measures can be used to evaluate a system over time or under different conditions. Two models by which routing flexibility can be assessed are presented. The resulting measure is the maximum contribution in monetary terms which is easily interpreted. These models incorporate factors such as reliability of the machines and the capacity available for production.

Machine flexibility is assessed by the minimum set-up time required to produce a given set of parts. The utility of this measure could be seen in choosing a design or in deciding the portfolio of part types to be produced on a given system. This measure clearly shows that flexibility is a function of operational considerations such as sequencing of parts and the positioning of tools. It also shows what impact such operational decisions have on the plant finances. It should noted, however, that the measures given here are partial costs or benefits and that once these measures as developed they should be used in a larger framework for technology selection.

One could extend the work on machine flexibility by simultaneously determining both the part sequence and the sequence of tools. This obviously makes the problem considerably more complex. Clearly, further work is required to strengthen the link between operational measures of a manufacturing system and the corresponding impact on a firm's financial status. Without models defining such links, managers will continue to have difficulty assessing investments, which contribute to a firm's manufacturing flexibility. The benefits of increased flexibility (both RF and MF) include decreased lead time and work-in-process inventories. These benefits could, in turn, provide increased market share and thereby they should be considered in a competitive framework to evaluate the strategic position of the firm. The short-run benefits or costs must be computed for the multitude of situations that could occur over the life time of the design.

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