CHOICE OF MOTHER WAVELET FOR IMAGE COMPRESSION

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ABSTRACT

Abstract- Image compression has been a major concern in the realm of communication especially for low data rate services. For the purpose of compression, various frequency domain analysis techniques have been explored, among which wavelet transform (WT) is quite popular. The process involves decomposition of the image using Multi Resolution Analysis (MRA) followed by thresholding and then encoding. The prime objective of this paper is proper selection of mother wavelet during the transform phase to compress the image so as to improve the quality as well as the compression ratio remarkably. The performance of the technique is measured in terms of percentage of zeroes (PERF0), percentage of energy retained (PERFL2), compression ratio (CR) and reconstruction error.

Keywords*-Wavelet ,Compression ,MRA ,PERF0 ,PERFL2*

1. INTRODUCTION

Visual communication has a tremendous impact on human lives. Multimedia is becoming increasingly important with application in several areas such as communication, transmission, storage of remote sensing images, education and medical images. Image compression methods that employ pyramid encoding, wavelet transforms have been successful in providing high rates of compression while maintaining good image quality. Discrete wavelet transform (DWT) has emerged as a popular technique for image coding applications; DWT has high decorrelation and energy compaction efficiency. This paper describes novel technique to extract the information from the original image and by exploring human visual interaction characteristics carefully, the compression algorithm can discard information which is irrelevant to human eye. By an image, we shall mean a digitized grey scale picture that consists of 2m by 2m pixels each of which takes a value between 0 and 2n-1. We shall denote the value of the pixel in row j1 and the column j2 of the image be Pj, j = (j1, j2). We view the image compression problem as one of approximating f by a second (compressed) function f. The object of such a compression algorithm will be to represent certain classes of pictures with less information than was used to represent the original picture. For a lossless algorithm, the original and compressed images will be the same, the error between then will be zero. Secondly, this paper presents that by selecting a type of wavelet family we can achieve better compression ratio,perf0 and perfl2 and energy retained in the image. In 2-DWT, perf0 and perfl2 are used to describe L2-norm recovery and compression score in percentage.

When compressing with orthogonal wavelets the energy retained is:

$$100^*\{\text{vector - norm(coeffs of the current decomposition)}\}^2/\{\text{vector - norm(original signal)}\}^2 \quad ....1$$

The number of zeros in percentage is defined by:

$$100^* \text{ (number of zeros of the current decomposition)/(number of coefficients)} ....2$$

To change the energy retained and number of zeros values, a threshold value is changed. Thresholding can be done globally or locally. In practice, compressing an image requires a 'trade-off' between compression and energy loss.

Energy loss per percentage zero = (3)

$$\frac{100 - \text{percentage energy retained}}{\text{percentage zeros}}$$

This gives a measure of how much energy is lost by compressing the images, so a smaller value for a ratio would mean the compression is less lossy, and therefore better.

This paper is organized as follows. In section 2-discuss the mathematical overview of an image ,section 3-discrete wavelet transform and Multiresolution analysis ,section 4-test cases and experimental results ,section 5-observation table and in section 6-conclusion of the paper is given.

MATHEMATICAL VIEW OF WAVELET ANALYSIS

Dilations and translations of the "Mother function" or "Analyzing Wavelet" Ø (x) define and orthogonal basis as :

$$\varphi_{(s,l)}(x) = 2^{s/2}\varphi(2^{s}x - l) \quad (4)$$

The variable s and l are integers that scale and dilate the mother function Ø to generate wavelets such as Daubenchies wavelet family. To span our data domain at different resolutions, the analyzing wavelet is used in a scaling equation as:

$$W(x) = {}_{k=\ 1}^{N\ 2}(1)^{k} C_{k+1}\varphi(2x + k) \quad (5)$$

where W(x) is the scaling function for the mother function Ø and Ck are the wavelet coefficients. The wavelet coefficients must satisfy linear and quadratic constraints of the form:

$$\sum_{k=0}^{N-1} C_k = 2 \ , \ \sum_{k=0}^{N-1} C_k C_{k+2l} = 2\,\delta_{l,0} \quad (6)$$

The coefficients {C0, C1............. Cn } act as a filter. The filter or coefficients are placed in a transformation matrix that is applied to a raw data vector. The coefficients are ordered using two dominant patterns, one that works as a smoothing filter like a moving average and one patterns works as data's "details" information. These two orderings of the coefficients are called a qaudrature mirror filter pair in signal processing parlance[2].

DAUBENCHIES WAVELET FILTER COEFFICIENTS

If we view a particular set of wavelets by Daubenchies such that if includes members ranging form highly localized to highly smooth coefficients. The simplest member DAUB4 has only four coefficients C0, C1, C2, C3.

Transformation matrix acting on a column vector is given as

$$\begin{bmatrix} c_0 & c_1 & c_2 & c_3 & & & & & & \\ c_3 & -c_2 & c_1 & -c_0 & & & & & & \\ & & c_0 & c_1 & c_2 & c_3 & & & & \\ & & c_3 & -c_2 & c_1 & -c_0 & & & & \\ \vdots & \vdots & & & & & \ddots & & & \\ & & & & & & c_0 & c_1 & c_2 & c_3 \\ & & & & & & c_3 & -c_2 & c_1 & -c_0 \\ c_2 & c_3 & & & & & & & c_0 & c_1 \\ c_1 & -c_0 & & & & & & & c_3 & -c_2 \end{bmatrix} \quad (7)$$

It is useful to think of the filter C_0, C_1, C_2, C_3 as smoothing filter H and the filter C_3, $-C_2$, C_1, $-C_0$ as non smoothing filter G. Infact C's are chosen to make G yield a zero response to a sufficiently smooth data vector.It is possible to reconstruct the original data vector of length N as N/2 smooth or S-components as N/2 detail or d-components. That is affected by requiring the matrix (7) to be orthogonal so that its inverse is just the transposed matrix.

WAVELET IMAGE CODING

It is well known that wavelet image coding consists of three main parts: Wavelet transform, quantization and redundancy removal. Most wavelet based signal compression systems are based on the structure shown in figure 1.The wavelet coefficients Ci are quantized and the resulting indices Ĉi are encoded without loss by the entropy encoder which usually employs contextual information [4].

Fig. 1: Simplified block diagram of a typical wavelet signal coder

WAVELET TRANSFORM SCHEME

In this study we work only with 2-D separable wavelet transform in which we associate to each image a function f(x) that is independent of the transform being applied. If we apply Haar transform or any other transform whose terms can be interpreted as being constant on square sub domains of pixels, we shall associate to the image the function f(x) defines for x : = (x1, x2) in I by:

$$f(x) := p_j \quad \text{for} \quad \frac{j_1}{2^m} \leq x_1 < \frac{j_1+1}{2^m} \qquad (8)$$

$$\text{and} \quad \frac{j_2}{2^m} < \chi_2 < \frac{j_2+1}{2^m} \qquad (9)$$

In this manner, wavelet transform is associated with each set of pixel values such that the function :

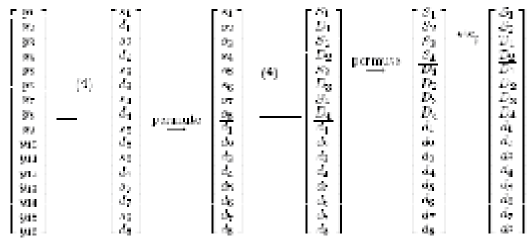
$$f = \sum_{k\geq 0, j=(j1,j2)} C_{j,k}\, \emptyset_{j,k} \qquad (10)$$

$$f = \sum C_{j,k}\, \emptyset_{j,k} \qquad (11)$$

DISCRETE WAVELET TRANSFORM AND MULTIRESOLUTION ANALYSIS

Wavelet Transform

The DWT consists of applying a wavelet coefficient matrix like (7) hierarchically, first to the full data vector of length N, then to the "smooth" vector of length N/2, then to the "smooth smooth" vector of length N/4 and so on until only a trivial number of "smooth...........smooth" components remain. This procedure is a pyramidal algorithm and the transformation matrix (7) obtained can result to the following matrix[3]:



DWT is a fast linear operation that operates on a data vector whose length is an integer power of two, transforming it into a numerically different vector of same length. The endpoint will always be a vector with two S's and a hierarchy of D's, D's, d's etc. A value of d of any level is termed as "wavelet coefficient" of the original data vector, the final values S ,S should strictly be called "mother-function-coefficients".

Multiresolution Analysis

WT performs Multiresolution image analysis (Mallat, 1989)[1]. The first component to multi resolution analysis is vector spaces. For each vector space, there is another vector space of higher resolution until you get to the final image. Also, each vector space contains all vector spaces that are of lower resolution. The basis of each of these vector spaces is the scale function for the wavelet. We can consider an image a vector space such as Vj would be perfectly normal image and Vj-1 would be that image at a lower resolution until we get Vo where there is only one pixel in the entire image. For such vector space Vj there is an orthogonal compliment called Wj and the basis function for this vector space is the wavelet. The subspace Vj are nested which implies Vj ∈ Vj+1. It is possible to decompose Vj+1 in terms of Vj and Wj.

$$V_J \text{Å} W_J = V_H - 0 \qquad (13)$$

$$\text{Also,} \;\; Wj \in Vj \qquad (14)$$

Ψ (x) Wo obeys translation property such that Ψ(x-k) Wo, k z. form a basis function for space Wo which is termed as wavelet function or mother function. DWT scaling function for 2-D DWT can be obtained by multiplying two 1-D scaling function; . Wavelet function for 2-D DWT can be obtained by multiplying two wavelet functions. For 2-D case there exists there wavelet functions that scan details in horizontal $\Psi(x, y) = \Psi(x)$, vertical $\Psi(x, y) = \Psi(y)$ and diagonal direction; $\Psi(x, y) = \Psi(x)\Psi(y)$. As a result, there are three types of detailed images for such resolution; horizontal, vertical and diagonal. Thus, a typical 2-D DWT used in image compression will generate hierarchical pyramid structure. Figure 2 shows two levels of wavelet decomposition image "peppers".

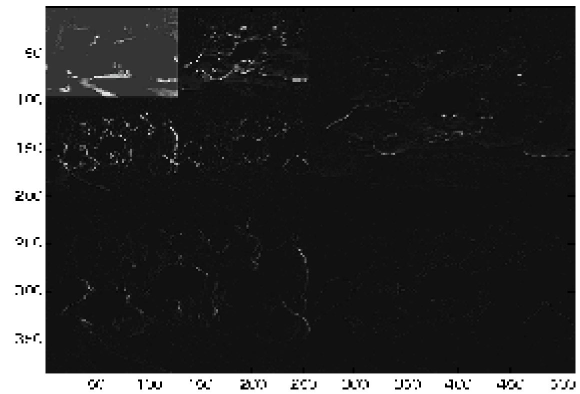


Fig. 2: Two level Decomposition

ALGORITHM FOR IMAGE COMPRESSION

Modern image compression techniques use the wavelet transform for image compression. We suggest the novel technique which is based on wavelet based on threshold Entropy with lossy enhanced run-length encoding. This method reduces the time complexity of wavelet decomposition and selects the sub bands, which include significant information in compact. The threshold entropy criterion funds the information contains of transform coefficients of sub bands. Threshold entropy is obtained by :

$$\sum_{i=0}^{N-1} | Xi | > threshold$$

The information contains of decomposed components of wavelet packets may be greater than or less than the information contain of component that has been decomposed. The sum of cost (threshold entropy) of decomposed components (child nodes) is checked with cost of component that has been decomposed (parent node). If the sum of the cost of the child nodes is less than the cost of parent mode, then the child nodes are considered as leaf nodes of the tree, otherwise child odes are neglected from the tree and parent node becomes leaf node of the tree. This process is iterated up to the last level of decomposition. The algorithm of best basis selection based on the threshold entropy is:

- Load the image.
- Set the current node equal to input image.
- Decompose the current node using wavelet packet tree.
- Evaluate decomposed components.
- Select proper mother wavelet, decomposition level and the threshold value to obtain the best possible compression ratio.

Results are obtained in terms of percentage of zeroes, percentage of energy retained and energy loss per percentage zero ratio and compression score.

TEST CASES AND EXPERIMENTAL RESULTS

We have taken the results of standard images bone, skull and skeleton. We have used mother wavelet families such as daubenchies, coiflets and biorthogonal. Results are observed in terms of percentage of zeros, percentage of energy retained, and energy loss per percentage zero ratio and compression score. The best results are presented in paper. Fig. 3: shows the original image of bone with its compressed and reconstructed image. Fig. 4: shows the original image, compressed image and reconstructed image of skull. Fig. 4: shows the original image, compressed image and reconstructed image of ske

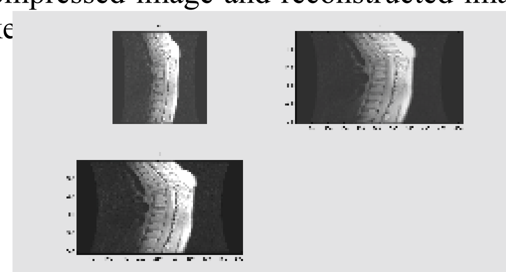


Fig. ... ge and reco...

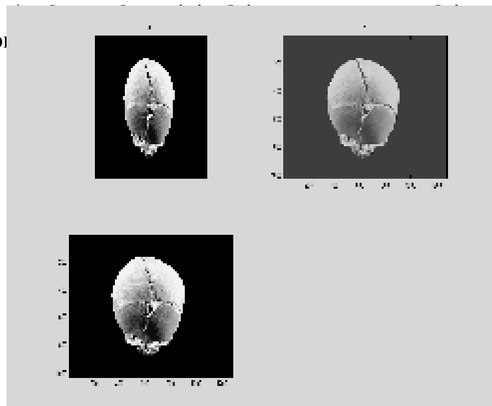


Fig. 4: shows the original image, compressed image and reco...

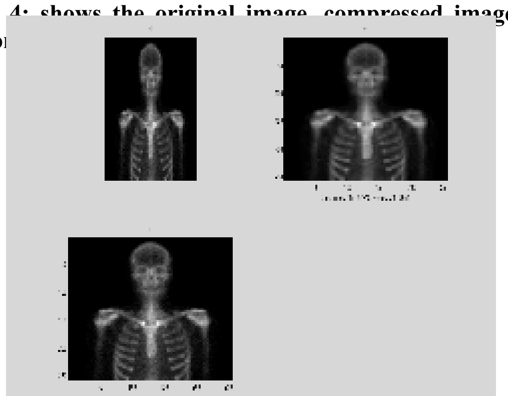


Fig. 5: shows the original image, compressed image and reconstructed image of skeleton.

OBSERVATION TABLE

Image	Wavelet Family	Perf0	Perfl2	Compression Score
bone	Db8	89.8	99.9	9.84
	Db10	89.3	99.9	9.36
	Coif1	91.5	99.8	11.77
	Coif5	87.6	99.9	8.08
	Bior2.6	91.4	99.9	11.73

Table 1: Perf0,Perfl2 and Compression score for image "bone"

Image	Wavelet Family	Perf0	Perfl2	Compression Score
skeleton	Db8	89.0	99.7	9.10
	Db10	87.8	99.7	8.22
	Coif1	93.0	99.5	14.4
	Coif5	83.7	99.8	6.16
	Bior2.6	90.5	99.6	10.57

Table 2: Perf0,Perfl2 and Compression score for image "skull"

Image	Wavelet Family	Perf0	PERFL2	Compression Score
skull	Db8	96.2	99.8	26.3
	Db10	96.0	99.8	25.0
	Coif1	96.7	99.8	30.9
	Coif5	95.4	99.9	21.8
	Bior2.6	96.4	99.9	28.3

Table 3: Perf0,Perfl2 and Compression score for image "skeleton"

CONCLUSION

In this paper selection of mother wavelet on the basis of image has been presented. Extensive result has been taken based on different wavelets. The result demonstrates that there is more compaction of energy into the approximation signal during wavelet analysis of biorthogonal family, hence describes more energy is retained or less energy can be lost during compression as well as the reconstruction error is lesser. It shows that bior2.6 represents lesser loss of information during the compression hence better is the quality of the image.

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